

Dispersive effects in neutron matter superfluidity

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Abstract. The explicit energy dependence of the single particle self-energy (dispersive effects), due to short range correlations, is included in the treatment of neutron matter superfluidity. The method can be applied in general to strong interacting fermion systems, and it is expected to be valid whenever the pairing gap is substantially smaller than the Fermi kinetic energy. The results for neutron matter show that dispersive effects are strong in the density region near the gap closure.

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1. Introduction

Neutron and nuclear matter superfluidity is one of the main issue in the physics of neutron stars. Superfluidity is expected to play a major role in some of the most striking phenomena occurring in neutron stars, like glitches and post-glitches transients [1], vortex pinning [2], neutron star cooling, and maybe strong magnetic field penetration [3]. However, since the observational data are only indirectly related to superfluidity, or need explicit models for their interpretation, a firm theoretical prediction of the superfluidity strength, based on microscopic *ab initio* calculations, appears highly required. Unfortunately, neutron and nuclear matter are strongly correlated systems, where short range correlations dominate the overall interaction energy, even at densities well below the saturation value. The superfluidity problem turns out, therefore, to be a complex many-body problem, where the delicate balance between short range interactions and the long range pairing correlations needs an accurate treatment. This problem was firstly considered in the works of ref. [4], within the variational Jastrow method. The medium effect on the effective pairing interaction was investigated in refs. [5, 6]. The correlation effects were treated in ref. [7] within a generalization of the Babu-Brown approach [8] to the effective nucleon-nucleon (NN) interaction and in the weak coupling limit. In general, these microscopic approaches seem to indicate a reduction of the pairing gap due to the medium, with respect to BCS approximation with the bare interaction. The use [9] in the BCS scheme of realistic bare nucleon-nucleon interactions, which reproduce the experimental phase shifts, can be a good starting point for a more sophisticated many-body treatments, and the connection between the pairing gap value and the phase shifts has been elucidated, in general, in ref. [10].

In all these microscopic approaches the single particle spectrum is usually considered within the effective mass approximation, or taken from normal state calculations. Dis-

persive effects, due to the energy dependence of the single particle self-energy, are usually neglected, or considered only in the weak coupling limit, and no first principle scheme to include them in the general gap equation has been proposed. Only recently [11] a self-consistent scheme has been developed, where the short range correlations and the pairing problem are treated on the same footing. The method is numerically complex, and it has been solved only for a schematic interaction [11]. It would be desirable to understand in simple terms the effect of the self-energy dispersion on the pairing strength, also in the case of strong coupling as in neutron matter. In this letter we present a general scheme for including dispersive effects in the gap equation, which is simple and accurate, provided the gap is substantially smaller than the Fermi kinetic energy. We then apply the method to neutron matter superfluidity with realistic interaction, and we show numerically that the size of the effect can be large in the vicinity of the gap closure.

2. Including self-energy in the gap equation

The general many-body theory of pairing in fermion systems has been formulated just after the BCS [12] solution has been introduced, and it can be found in standard textbooks [13, 14]. The main framework is the Green's function (GF) formalism, which generalizes the Gorkov's method beyond the BCS approximation. The single particle Green's function \mathcal{G} has a 2x2 matrix structure, with normal diagonal components F_1 and abnormal off-diagonal components F_2 ,

$$\mathcal{G}(k, \omega) = \begin{pmatrix} F_1(k, \omega) & F_2(k, \omega) \\ F_2(k, \omega) & -F_1(k, -\omega) \end{pmatrix}, \quad \mathcal{G}^{-1}(k, \omega) = \begin{pmatrix} \tilde{\epsilon}_k - \omega + M(k, \omega) & \Delta(k, \omega) \\ \Delta(k, \omega) & -(\tilde{\epsilon}_k + \omega + M(k, -\omega)) \end{pmatrix} \quad (1)$$

In the expression for the inverse Green's function \mathcal{G}^{-1} , we have introduced the quantity $\tilde{\epsilon}_k = \hbar^2 k^2 / 2m - \mu$ as the single particle kinetic energy, with respect to the chemical potential μ , the diagonal single particle self-energy $M(k, \omega)$ and the momentum and energy dependent gap function $\Delta(k, \omega)$. Here we are assuming s-wave singlet pairing, and therefore we omit spin indices. They simply express the coupling between the time-reversal states (k, \uparrow) and $(-k, \downarrow)$. Both $M(k, \omega)$ and $\Delta(k, \omega)$ can be expanded in terms of the NN interaction and the full GF itself, which in general entails a self-consistent procedure. The gap function $\Delta(k, \omega)$, however, is solution of the homogeneous generalized Bethe-Salpeter equation [13], and as such it satisfies the generalized gap equation

$$\Delta(k, \omega) = \sum_{k'} \int d\omega' \frac{I(k\omega, k'\omega') \Delta(k', \omega')}{(\tilde{\epsilon}_{k'} - \omega' + M(k', \omega'))(\tilde{\epsilon}_{k'} + \omega' + M(k, -\omega')) + \Delta(k', \omega')^2} \quad (2)$$

where $I(k\omega, k'\omega')$ is the irreducible NN interaction at zero total energy and momentum. If one takes the bare NN interaction for the interaction I , and the Hartree-Fock approximation for the diagonal self-energy $M(k, \omega)$, the standard BCS approximation is recovered. It has to be noticed that the energy dependence of the gap function $\Delta(k, \omega)$ originates only from the energy dependence of the irreducible interaction I . In fact, if the interaction is taken as energy-independent, the gap function is also energy-independent, despite the possible energy dependence of the self-energy $M(k, \omega)$. Since we are looking for an

estimate of the dispersive effects, we indeed assume the irreducible interaction as energy independent, while we keep the full energy dependence of the self-energies. Then, the energy integration appearing in the gap equation (2) can be performed with a good accuracy in the limit of small self-energy imaginary part, since then the main contribution is expected to come from the poles close to the real axis. The denominator is an even function of the energy ω (we remind again that single particle energies are measured with respect to μ), and, therefore, the kernel presents two poles, symmetrical with respect to the origin in the complex ω -plane. This is a feature typical of the superconducting phase. Formally, the pole energies $\pm E_k$ are the solutions of the implicit equation

$$\pm E_k = \frac{1}{2}(M(k, \pm E_k) - M(k, \mp E_k)) \pm \sqrt{\left[\tilde{\epsilon}_k + \frac{1}{2}(M(k, -E_k) + M(k, E_k))\right]^2 + \Delta(k)^2} \quad (3)$$

If the energy dependence of $M(k, \omega)$ is neglected, than Eq. (3) reduces to the usual square root expression for the quasi-particle excitation energy of the BCS approximation. On the other hand, in the non-superconducting limit $\Delta \rightarrow 0$, and neglecting the imaginary part of $M(k, \omega)$, one can verify that Eq. (3) reduces to the usual self-consistent equation, e.g. Brueckner [15], for the single particle energy e_k

$$e_k = \tilde{\epsilon}_k + M(k, e_k) \quad (4)$$

Equation (4) is valid whenever Δ is negligible, in particular for momenta far away from the Fermi surface, since then $|\Delta_k| \ll |\tilde{\epsilon}_k|$. We now make use of the assumption of a gap Δ smaller than the normal self-energy, which is mainly determined by short range correlations. The size of the normal self-energy is indeed of the same order of the Fermi kinetic energy E_F . In this case, Eq. (4) will be valid to order $\Delta(k_F)/E_F$, and therefore on the right hand side of Eq. (3) we can replace E_k with e_k , solution of Eq. (4), to get

$$E_k \approx \frac{1}{2}(M(k, |e_k|) - M(k, -|e_k|)) + \sqrt{\left[\tilde{\epsilon}_k + \frac{1}{2}(M(k, -e_k) + M(k, e_k))\right]^2 + \Delta(k)^2} \quad (5)$$

where the self-energy is now calculated in the normal phase. The procedure is justified, provided $M(k, \omega)$ is a smooth function of ω . It will be further discussed below.

The residue R of the kernel at each one of the pole can be easily calculated

$$R = \left[1 - \frac{1}{2}((1 - \Theta_k)a_k + (1 + \Theta_k)b_k)\right]^{-1} \quad (6)$$

$$\Theta_k = \frac{\tilde{\epsilon}_k + \frac{1}{2}(M(k, -E_k) + M(k, E_k))}{E_k} \quad (7)$$

$$a_k = \left(\frac{\partial M}{\partial \omega}\right)_{\omega=E_k} ; \quad b_k = \left(\frac{\partial M}{\partial \omega}\right)_{\omega=-E_k} \quad (8)$$

In the limit $\Delta \rightarrow 0$, Eq. (6) is the usual expression for the quasi-particle strength Z_k , provided the momentum k is close enough to k_F . The corrections to the normal phase value of $1 - R^{-1}$ are of the order $\Delta(k_F)/E_F$, and therefore the residue R can be

identified with Z_k , at least in the vicinity of the Fermi momentum. Far away from the Fermi momentum, the residue R has still the expression of Eq. (6) in the limit of small imaginary part, despite the quasi-particle concept becomes less meaningful, since its width can be large (but it can be still much smaller than the real part of the energy). In this case the procedure is just an accurate method of calculating the energy integral (pole approximation). For simplicity, the residue will be denoted by Z_k in all cases. Within these approximations, the generalized gap equation (2) reads

$$\Delta(k) = - \sum_{k'} I(k, k') Z_{k'} \frac{\Delta_{k'}}{2 \sqrt{\left[\tilde{\epsilon}_k + \frac{1}{2} (M(k, -e_k) + M(k, e_k)) \right]^2 + \Delta(k)^2}} \quad (9)$$

Eqs. (5), (6), (9) contain the main result of the paper. It has to be noticed that in the generalized gap equation (9) the square root in the denominator does not coincide with the quasi-particle energy of Eq. (5), in contrast with the usual BCS approximation. This feature is general and it is not bound to the approximation of Eq. (5). The approximation of Eq. (5) to the expression inside the square root is valid up to corrections of order Δ/E_F . They can be absorbed in large part by a small shift of the chemical potential μ .

Before going to the application of the formalism to neutron matter, let us discuss Eq. (9) in the extreme weak coupling limit, where one assumes that the main contribution to the momentum integral is concentrated around the Fermi surface. In this limit, following the standard procedure of expanding the integrand of Eq. (9) around k_F , one gets [16]

$$\Delta_F = 8 \frac{E_F}{m_F} \exp\left(-\frac{1}{m_F Z_F \pi^2 n_0 I(k_F)}\right) \quad (10)$$

where n_0 is the density of state for the free Fermi gas and m_F the so called $k - mass$ (in units of the bare mass) [17]. The interaction $I(k_F)$ is the diagonal matrix element of the NN potential in the considered channel (1S_0 for neutron matter), in the plane wave representation. The self-energy effects are, therefore, contained mainly in the factor $m_F Z_F$, which can be written also as $m^* Z_F^2$, since the full effective mass $m^* = m_F / Z_F$ [17]. This is the standard result for the weak coupling limit [18]. Eq. (9) generalizes the treatment to the case where the contribution from momenta far from the Fermi momentum is relevant. The appearance of the $k - mass$ is a peculiar feature of the pairing phenomenon, and it is a direct consequence of the coupling between time-reversal states. In Eq. (9) the combination $M(k, -\omega) + M(k, \omega)$ gives rise to the *combined* density of state of the pair $\{(k, \omega); (-k, -\omega)\}$, which is mainly determined by the $k - mass$.

The weak coupling limit is not valid in general for neutron or nuclear matter [15], if one starts from the bare NN interaction. This can be seen directly from the observation that often the gap equation has a well defined solution even when the interaction matrix element $I(k_F)$ is positive. This is due to the dominant role of the off-diagonal matrix elements $I(k, k')$. Therefore, one must solve the more general equation (9) in this case. The above considerations are, anyhow, still valid.

3. Application to neutron matter superfluidity

In order to estimate the dispersive effects on the superfluid gap of neutron matter, we have solved Eq. (9), with the bare Argonne v_{14} potential as the pairing interaction $I(k, k')$ and with the self-energy calculated in the Brueckner approximation at the lowest order, see Fig. 1a, with the same interaction. The higher order contribution of Fig. 1b turns out to be indeed negligible in the relevant density range. In the superfluid phase, in principle, the diagonal self-energy $M(k, \omega)$ differs from the self-energy in the normal phase. The main contribution not present in the normal phase originates from the coupling of the single particle motion with the superfluid collective modes. The latter correspond mainly to the center of mass motion of the Cooper pairs and their possible “vibrations” [19, 20]. The branch starting at zero energy, in the long wave-length limit, is the branch of the Goldstone boson [19], corresponding to the gauge invariance symmetry breaking at the superfluid phase transition. This contribution to the diagonal single particle self-energy is expected to be at most of the order of the superfluid condensation energy per particle, and therefore negligible with respect to the typical short range correlation energy, as calculated e.g. in Brueckner theory, at least to the extent that $\Delta/E_F \ll 1$. For the same reason, the deviation of the occupation number from the free gas value and the presence of a forbidden energy region, of order Δ , around the Fermi energy, typical of the pairing phenomenon, seem to play no relevant role in determining the size of the self-energy. It appears, therefore, justified to adopt for $M(k, \omega)$ its normal phase value.

The choice of the bare interaction for $I(k, k')$ is suggested by the observation that no ladder summation should be included in the irreducible interaction kernel $I(k, k')$ [18, 15]. Of course, other terms, like polarization diagrams, should be included [6], but here we want simply to single out the dispersive effects, and therefore it appears meaningful to compare the results obtained with and without self-energy, within the same scheme of approximation.

In Fig. 2 is reported, for $k_F = 0.9 fm^{-1}$, the imaginary part of the neutron self-energy $M_I(k, e(k))$ at the quasi-particle pole, as a function of the momentum k , together with the real part of the quasi-particle energy $e(k)$ (calculated with respect to μ). One can see that indeed the imaginary part is small with respect to the real part. The situation is completely similar for the other densities. The residue Z_k of Eq. (6), which appears in Eq. (9) for the gap function, is reported in Fig. 3 for three densities. According to Migdal-Luttinger theorem [18], the value Z_F of Z_k at $k = k_F$ is the discontinuity of the momentum distribution at the Fermi momentum (in the normal phase). One must have, therefore, $0 < Z_F < 1$. One can see, however, that Z_k exceeds 1 slightly in some interval well above k_F . This is not surprising, since for large ω values, at fixed k , the real part of the self-energy is an increasing function of ω , and it approaches asymptotically an energy-independent value [22] (and therefore $Z_k \rightarrow 1$). As already mentioned, for large momentum the pole approximation is just an accurate method of calculating the relevant energy integral. The position of the pole is, of course, not exactly on the real axis, but numerical estimate of the second derivative of the self-energy shows that to calculate Z_k on the real axis is an extremely good approximation. The factor Z_k is also related to the single particle occupation number $n(k)$. In fact, the contour integral,

closed in the lower complex plane, of the single particle GF equals $1 - n(k)$, $k > k_F$. Besides the pole, this integral receives contribution from the regular (non polar) part of the single particle GF [18]. If $Z(k) > 1$, this means simply that the regular part leads to a slightly negative contribution. We have checked, indeed, that the single particle spectral function, calculated with the same self-energy (including of course the imaginary part), satisfies the sum rules and gives well defined occupation numbers $n(k)$. A full account of the calculations will be reported elsewhere. Anyhow, this small deviation from 1 does not affect at all the final results, and one can take $Z_k = 1$ in this momentum region.

Finally, in Fig. 4 is reported the pairing gap at the Fermi energy as a function of density for three different cases : i) without self-energy [9], ii) with the Z_k factor in the numerator of the gap equation (9), iii) with both the self-energy in the denominator and the Z_k factor. The reduction of the pairing gap is substantial at the highest densities, near the gap closure.

3. Discussion and conclusion

We have developed a method to include the single particle self-energy in the gap equation, and in particular dispersive effects, beyond the usual BCS approximation. The method rely on the assumption of a small gap with respect to the Fermi kinetic energy and strong short range correlations, typical of the neutron matter in the inner crust of neutron stars. The results indicate that dispersive effects can strongly reduce the gap value near its closure. The effect is due both to the quasi-particle strength Z_k and to the $k - mass$, which enter in the generalized gap equation (9). This result appears in line with the work of ref. [11], where the self-consistent treatment of pairing and short range correlations seems indeed to reduce strongly the gap value mainly because of these two factors [11, 21]. Of course, before drawing any conclusion on the pairing strength in neutron matter, one should include, along a consistent scheme, the correlation effects on the irreducible interaction $I(k, k')$. Work in this direction is in progress.

The extension of the method to symmetric nuclear matter, possibly relevant for pairing in nuclei, appears problematic. In that case the approximation of a small imaginary part looks less justified away from the Fermi momentum [22]. Furthermore, the neutron-proton pairing in the ${}^3S_1 - {}^3D_1$ channel is too strong, i.e. $\Delta \sim E_F$, to be treatable in the proposed approximate method. In both cases a self-consistent procedure, involving both the self-energy and the effective interaction, seems to be the only viable method.

Figure captions

Fig. 1.- One (a) and two (b) hole-line diagrams contributing to the nucleon self-energy in the Bethe-Brueckner-Goldstone expansion. The wavy lines indicate Brueckner G-matrices.

Fig. 2.- Real part $e(k)$ and imaginary part M_I of the quasi-particle pole, as a function of the momentum k at the Fermi momentum $k_F = 0.9 fm^{-1}$.

Fig. 3.- The residue Z_k at the quasi-particle pole as a function of the momentum k at three values of the Fermi momentum k_F .

Fig. 4.- The superfluid gap value, at the Fermi momentum, as a function of density, in the case of free single particle spectrum (diamonds), with the inclusion of the factor Z_k (crosses) and with the inclusion of both Z_k factor and the self-energy in the single particle spectrum (squares).

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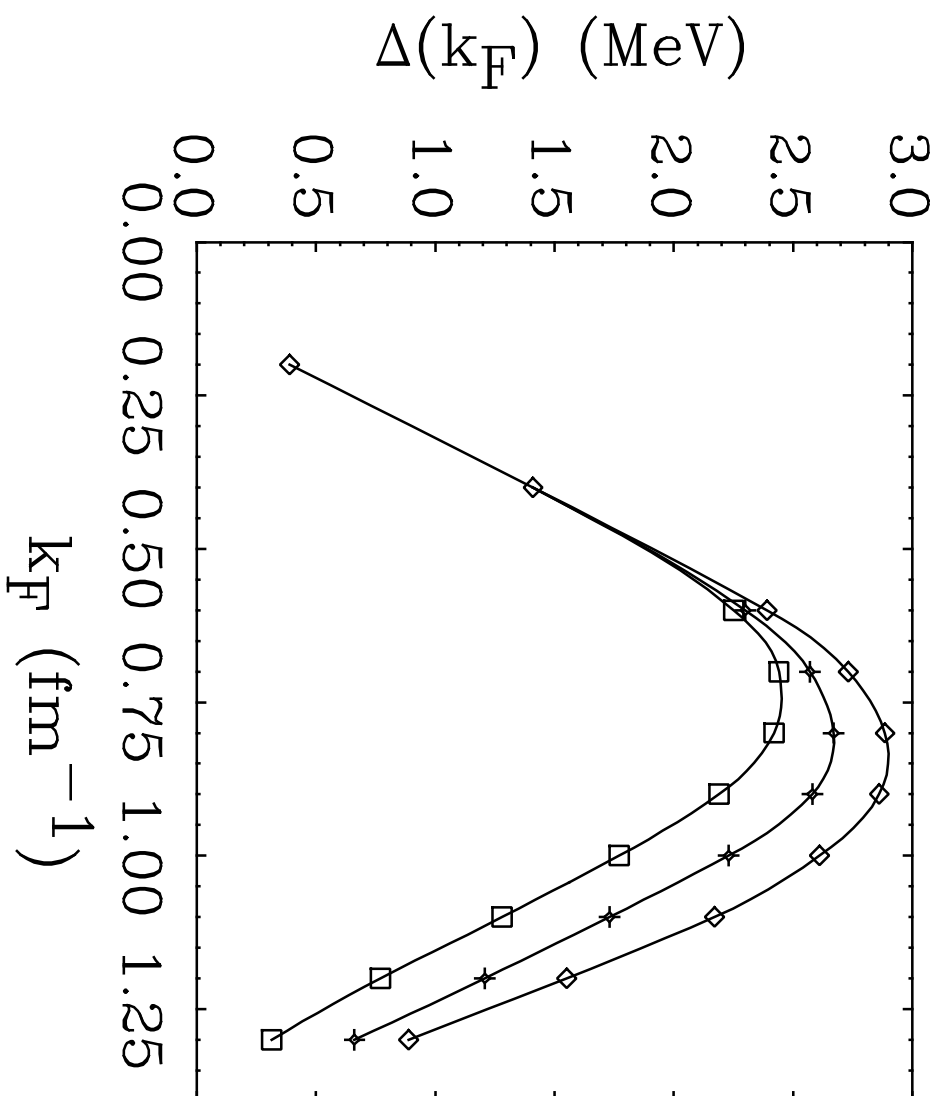


Fig. 4

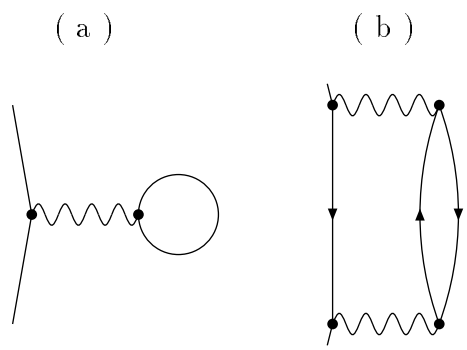


Fig. 1

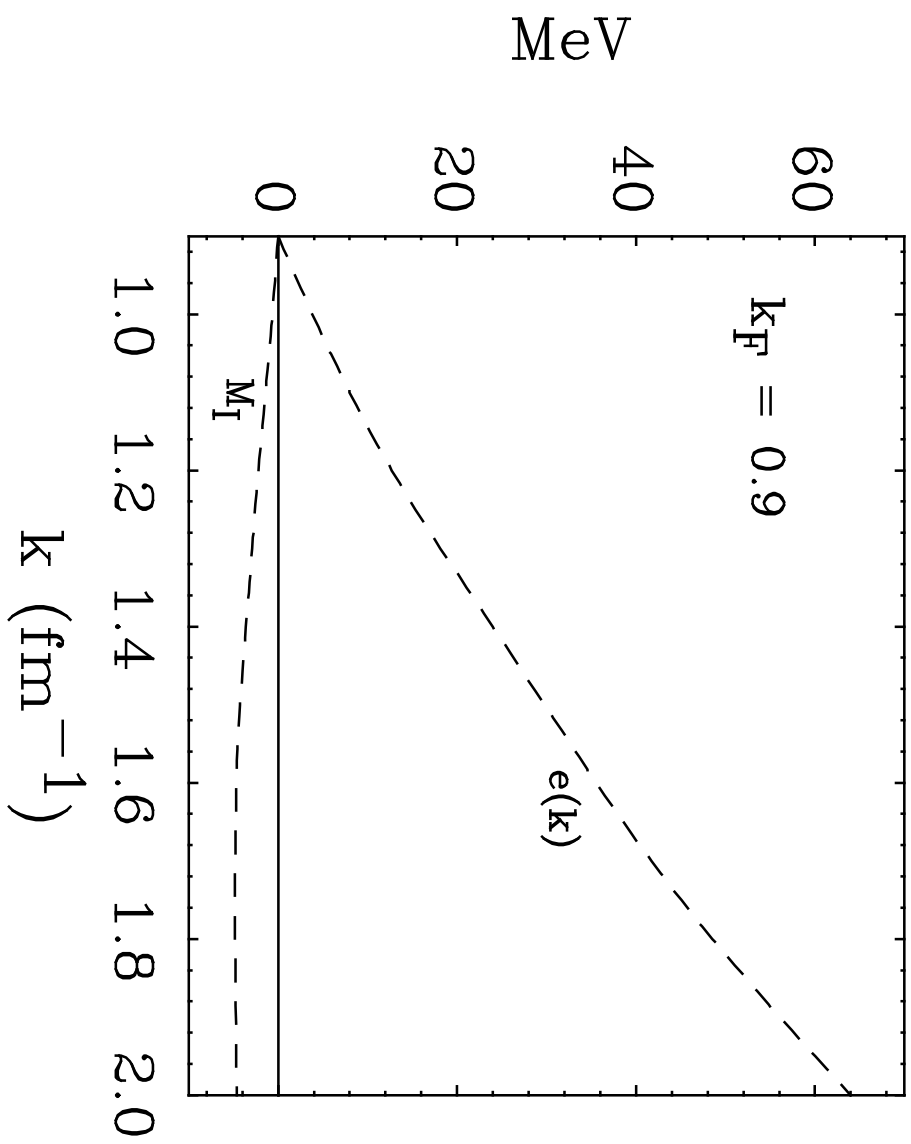


Fig. 2

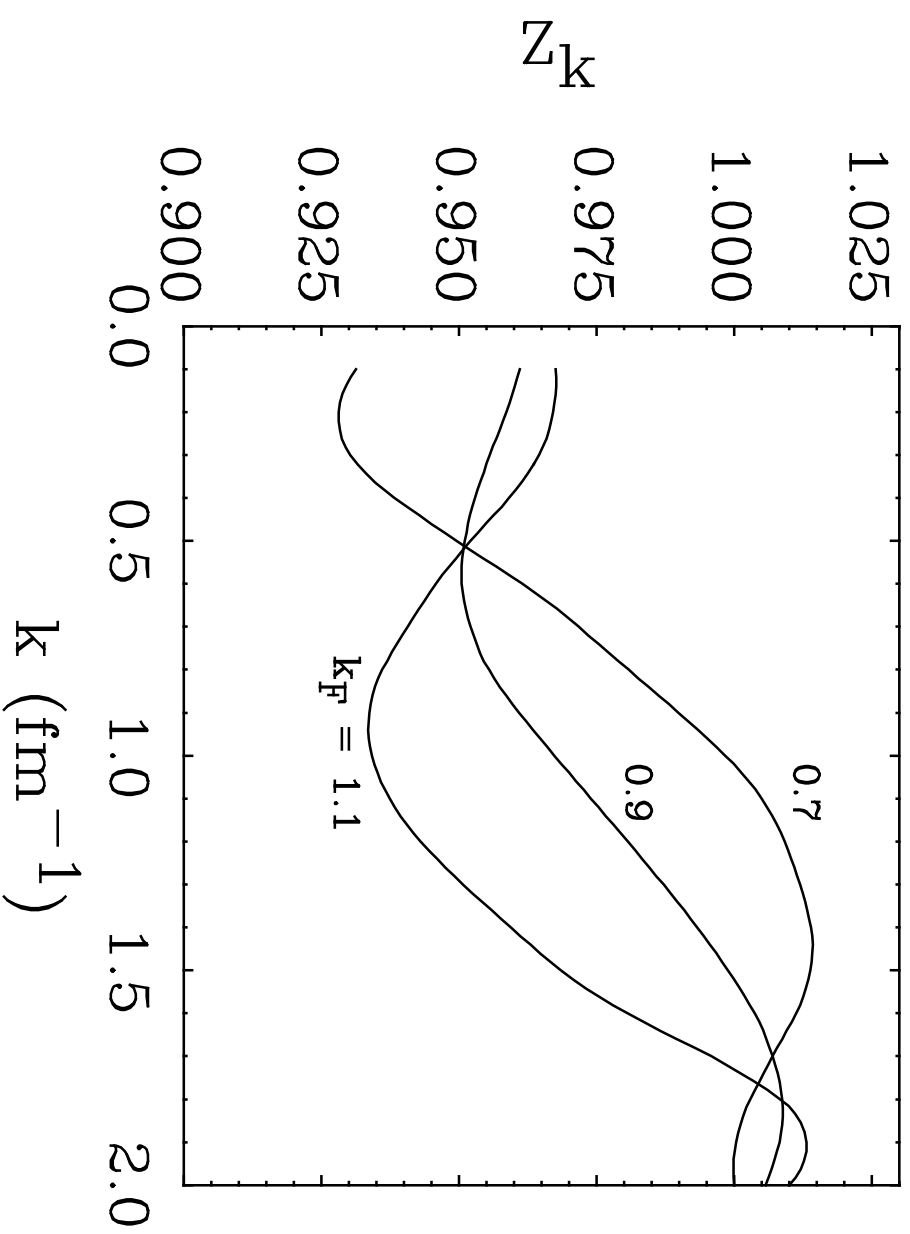


Fig. 3